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FINAL REPORT ON THE RESEARCH ACCOMPLISHED UNDER AFOSR CONTRACT NO. 87-0136 DURING THE PERIOD APRIL 1, 1987 - JULY 31, 1989

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1. SUMMARY OF WORK COMPLETED UNDER AFSOR CONTRACT NO. 87-0136..

The Gaussian model has been used to describe many random phenomena in science and engineering because of its versatility and mathematical simplicity. However, the Gaussian model is not universally applicable; and, in fact, there are many instances, both in the areas of theoretical research and engineering applications, where the need of non-Gaussian models, particularly those with infinite variance, can be identified. For instance, man-made noise in a hostile environment can be made to depart from Gaussian behavior; and the natural noise occurring in situations where weak signals need to be extracted also tends to not follow the Gaussian pattern. These are but a few examples that might be of particular interest to defense agencies, where non-Gaussian modeling and their analysis are most desirable. Keywords: Gaussian Models, Integral and Series Representation, Prediction, Sample Path, Statistical and Structural Properties, Non-mode noise, Statistical processes, White Noise, Acoustics (KR)

The analysis of Gaussian signal/noise models (≡ processes) is facilitated by the fact that the first and second order moments are sufficient to characterize completely their finite dimensional distributions. There are, at present, many varieties of techniques available in the literature which can be used to solve "most" engineering problems arising in a Gaussian environment. However, this methodology can not be adopted to the non-Gaussian situation, particularly, when the model process has infinite variance.

The long term goal of this proposal is to develop a comprehensive theory which allows one to study non-Gaussian signal/noise models (≡ processes), including those with *infinite variance.* The prime emphasis in the program is to pursue fundamental research with the aim to develop new methodologies and to use these for solving important problems in the areas of signal processing, estimation and prediction theory, statistical inference, sample path and structural analysis for the class of (non-Gaussian) infinitely divisible (i.d.) processes and certain non-linear functionals of such processes.

Towards achieving this long term objective, a unified approach for the investigation of some of these problems was proposed in the following three broad topics:

i. Integral and series representations for i.d. processes.

ii. Prediction for i.d. integral processes.

iii. Sample path, statistical and structural properties of i.d. integral processes.

We have made a significant and extensive progress towards achieving the research goal set out in our original proposal. In fact, in some cases, we were able to enlarge the scope of our research considerably by investigating broader aspects of the proposed problems and by providing definitive answers to these. We have already successfully completed all the works proposed in Part (i), and, in fact, we have settled several other interesting related problems which were not specifically noted in the proposal. In addition, we have completed substancial part of the works proposed in Parts (ii) and (iii). The remaining problems and those which arose during the course of actual research have been proposed as part of the renewal proposal for a period four additional years. Now we shall highlight our major research accomplishments under these three different topics.

The core works based on the research outlined in Part (i) are reported in a series of four papers [R.1-4], where Wiener type stochastic integrals are developed and the spectral and series representations for i.d. random vectors are obtained. These works provide important fundamental tools to continue research to work on the other two parts. Just as in the Gaussian setting, we envisage that the spectral and series representations obtained in [R.1-4] would play a significant role in the study of various problems (e.g., prediction, estimation, statistical inference, etc.) for i.d. processes. Further, because of the novelty of approach introduced and the generality of results obtained, these works also provide promising tools for studying challenging problems other than those proposed. The works [R.5] and [R.6] concern, respectively, with the multilinear forms in stable random vectors and the multiple stochastic integrals of vector functions relative to semistable random measures; and the work [R.7] is related to the problem of stochastic integrals of random functions relative to white noise. These works can be viewed as an outgrowth of the research performed in [R.1-4] where single stochastic integrals of deterministic functions

relative to i.d. noise and other related problems are considered. The possible fields to which the works presented in [R.5-7] have applications include statistics, quantum mechanics and nonlinear filtering theory. More detailed summaries of these seven works now follow.

The main result in [R.1] provides spectral representations for arbitrary discrete parameter (real) i.d. processes as well as for (centered) continuous parameter (real) i.d. processes which are separable in probability. It is shown that these representations are valid not only in law but also almost surely. Further, the relationship between the space generated by the representing functions and the linear space of the process is fully investigated. The main tools developed here and used for the proof of the representation theorems are (i) a "polar factorization" of an arbitrary Lévy measure on a separable Hilbert space, and (ii) the Wiener-type stochastic integrals of non-random functions relative to arbitrary "i.d. noise". The results obtained are very general and provide, in a unified way, all earlier known spectral representations for stable and semistable processes.

The results in [R.2] provide spectral representations for *complex* symmetric stable and semistable processes. It may be noted here that while analyzing linear problems for, say, α-stable processes, it is necessary to deal with complex α-stable processes even when one is interested only in real ones. One reason for this is that the spectral representation of a real stationary α-stable process has to be necessarily expressed in terms of a complex process. This explains the need and importance of the results obtained in [R.2].

Works [R.3-4] deal with the series representations of i.d. random vectors. First, a general result on the convergence and the distribution of certain series, which generalizes the shot noise processes, is proved. The result is then used to derive LePage-type series representations of i.d. random vectors in Banach spaces, that generalizes the works of several authors in this direction. This result, when specialized to i.d. stochastic processes, yields very useful series representations of these processes. In contrast with the Karhunen-Loève representation of Gaussian processes, the terms in our series are given explicitly in terms of the corresponding stochastic integral representation of the process (details of

this connection with integral processes are given in [R.8]). In addition, [R.4] also contains a complete characterization of the class of i.d. random vectors which can be represented by series which are conditionally Gaussian. We prove that such a class of conditionally Gaussian i.d. random vectors can be characterized in terms of two parameters (ϕ, V), where ϕ is a function with completely monotone derivative and V is an arbitrary measure (on a Banach space). This study has been motivated by the fact that stable processes have the above property; this property played a central role in several distinctive works on path continuity and extrema for stable processes. Therefore, one may expect that many interesting results can be proved for conditionally Gaussian i.d. processes and some of them are proposed to be studied in Section 4.

Decoupling inequalities give bounds for moments of multiple stochastic integrals by the respective moments of iterated coordinate-wise independent stochastic integrals. In [R.5] we find a natural condition that ensures decoupling inequalities for multilinear forms in stable random vectors. As an application of our result, we give an explicit condition for the finiteness of the moments of multilinear forms in stable motion. This work strengthens and complements an important result of de Acosta (1987).

Multiple stochastic integrals have been studied by many authors both for their intrinsic value as well as for their applications to other fields. In recent years several authors have developed multiple stochastic integrals of vector valued functions with respect to symmetric and nonsymmetric stable random measures. Using similar methods, multiple stochastic integrals of vector functions relative to more general semistable random measures are developed in the work [R.6]. This work also contains two additional results: One of these provides a comparison between the moments of multiple stochastic integrals relative to α-stable and α-semistable random measures; and the other shows that the class of vector integrable functions relative to an α-semistable random measure coincides with the class of vector integrable functions relative to a symmetric α-stable random measure.

In work [R.7], the large deviations for nonlinear transformations of white noise are

studied. In particular, an abstract version of Freidlin-Wentzel inequalities for solutions of stochastic differential equations is obtained which, in turn, is used to extend the Varadhan contraction principle.

Now we turn our attention to discussing research accomplished under topic (iii). We may also note here that at least two of the works discussed below are indeed related to the research proposed in Part (ii).

The results reported in [R.8-9] resolve several questions about the path properties of i.d. processes proposed under topic (iii). Path properties of processes represented in the form of multiple stable integrals are studied in [R.17]. The results reported in [R.10-11] are concerned with the study of the structure of supports of certain i.d. probability measures on linear spaces; and the works [R.12-13] and [R.16] are devoted to study the properties of the densities of i.d. probability measures on Euclidean spaces. The works [R.10-11], besides their intrinsic mathematical significance, also have applications in the study of path properties of i.d. processes and signal processing of such stochastic signals. For instance, the study of supports of measures induced by processes can be viewed as seeking sets which contain almost all sample paths yet which are as small as possible. Thus, in principle, this should help in proving equivalence and singularity of i.d. processes (see, e.g., [R.12]), which in turn could be used in signal detection and estimation problems for such stochastic signals. From this perspective these works are related to the research proposed both in topics (ii) and (iii). The results reported in [R.14] aim at the extension of the classical ergodic theorems of the commutative probability theory to the modern non-commutative setting; this work is related to some problems proposed in (ii). Finally in [R.15], using the series representations obtained in [R.4], a new proof of the zero-one laws for i.d. measures on Banach spaces is provided. In the following paragraphs, we summarize the main results reported in the above cited six research papers.

Sample path properties of i.d. stochastic integral processes are studied in [R.8]. It is shown that various properties of the sections of the deterministic kernel are inherited by

the sample paths of the stochastic integral process. In particular, the following properties are examined: boundedness, continuity, lack of oscillatory discontinuities, boundedness of the p-th variation, differentiability and integrability. As the main tool for the proofs, a series representation of symmetric stochastic integral processes is fully developed which complements the results reported in [R.3-4].

In work [R.9], we provide conditions for an i.d. process to have non-random oscillations. This result yields, immediately, important information about the paths of such processes; for example, one gets the Belayev type dichotomy between path continuity and unboundedness for stationary or self-similar processes. The main tool used here is a series representation for stochastic integral processes in the form developed in [R.8].

The works [R.10-11] are devoted to resolve two long-standing conjectures concerning the structure of the supports of certain i.d. probability measures on linear spaces. These works also yield an affirmative answer to the question, open for some time, of whether the support of an α-table probability measure, $1 \leq \alpha < 2$, on a separable Banach space B, is a translate of a linear space. A short summary of some of these results follows: Let μ be a centered i.d. probability measure on B; assume that M, the Lévy measure of μ, admits a polar decomposition. One of the main results states that if M integrates the norm on the unit ball U of B, then $S(\mu)$, the support of μ, is a translate of the closed convex cone generated by the support of the spectral measure σ of μ; and if M does not integrate the norm on U, then $S(\mu)$ is a linear space and is equal to the closure of the semigroup generated by the support of M and negative of the barycenter of σ. Extensions of these results to more general linear spaces (than B) are also provided.

The knowledge of the analytic properties of the densities of i.d. probability measures on R^d is an extremely valuable information both for theoretical as well as for practical considerations. Yet, with the exception of certain very special cases (e.g. Gaussian), very little is known about these properties of the densities of most other i.d. probability measures on $R^d, d \geq 2$. The major part of the works [R.12-13] is devoted to provide several

results regarding the analytic and "positivity" properties of the densities of i.d. probability measures on R^d. In one of these results a verifiable sufficient condition is provided which guarantees the existence, analyticity and positivity (on R^d) of the densities of certain probability measures on R^d. In another it is shown that, under a mild condition, a continuous density of an i.d. probability measure μ on R^d is always positive on the interior of the support of μ and is zero outside of it. This result extends, to a considerable degree, a similar result for α-stable probability measures due to Taylor and Kesten.

The main result reported in [R.14] provides, under suitable regularity conditions, an asymptotic formula for the ergodic averages for a normal operator acting on the L_2-space on a Von-Neumann algebra with a faithful normal state. This result, as noted earlier, is motivated with a view to obtain the analog of an ergodic theorem in the classical commutative probability theory to the modern non-commutative setting.

In report [R.15] we present an application of series representations from [R.4] to the zero-one laws for infinitely divisible measures on Banach spaces. We give also a simple criterion for the zero-one law in terms of the associated Lévy measures on the line. The same idea can be used to establish zero-one laws for integral processes which, together with Karhunen-Loève type representation of Gaussian processes, leads to a unified approach to zero-one laws for i.d. processes. The novelty of our approach is that, unlike the earlier works which treat these zero-one laws by using strictly algebraic and topological methods, we provide transparent probabilistic arguments. This is made possible mainly because we used the above noted series representations for i.d. random vectors for our proof for the zero-one laws.

In [R.16] we provide a new characterization of convex cone (with vertex at 0) in R^d; this work was motivated from and resolves a recent conjecture of S.C. Port and R.A. Vitale (1988). Using this characterization, we also provide, in [R.16], a new proof of the positivity properties of α-stable densities, $0 < \alpha < 2$, $\alpha \neq 1$, on R^d.

In [R.17] we study the sample path properties of stochastic processes represented in the

form of multiple symmetric α-stable integrals. Similarly as in [R.8], we demonstrate that properties of the deterministic kernel have a strong impact on the sample path properties of the process. We also investigate the zero-one laws for such sample path properties as well as the tail behaviour of the distribution of the suprema of such processes.

The works [R.16] and [R.17] are, at present, in the process of typesetting and will be send soon.

2. LIST OF PAPERS ACKNOWLEDGING AFOSR SUPPORT.

[1] Balram S. Rajput and Jan Rosinski. *Spectral representations of infinitely divisible processes.* To appear in Probability Theory and Related Fields, (1989).

[2] Balram S. Rajput and K. Rama-Murthy. *On the spectral representations of complex semistable and other infinitely divisible stochastic processes.* Stochastic Processes and Their Applications 26 (1987), 141-159.

[3] Jan Rosinski. *On series representations of infinitely divisible random vectors and a generalized shot noise in Banach spaces.* Preliminary report, (1987).

[4] Jan Rosinski. *On series representations of infinitely divisible random vectors.* To appear in Annals of Probability, (1989).

[5] Balram S. Rajput and Jan Rosinski. *Complements on decoupling inequalities for multilinear functions in stable random vectors.* To appear in Probability and Mathematical Statistics, (1989).

[6] Xavier Raja Retnam. *On multiple stochastic integral with respect to a strictly semistable random measure.* Ph.D. Dissertation, The University of Tennessee, (1989).

[7] Wlodzimierz Smolenski and Rafal Sztencel. *Large deviations for non-linear radovifications of white noise.* To appear in Lecture Notes in Mathematics, Springer Verlag, (1989).

[8] Jan Rosinski. *On path properties of certain infinitely divisible processes.* To appear in Stochastic Processes and Their Applications, (1989).

[9] Stamatis Cambanis, John P. Nolan and Jan Rosinski. *On the oscillation of infinitely divisible processes.* Submitted to Stochastic Processes and Their Applications, (1988).

[10] Balram S. Rajput. *On the geometric structure of the support of stable index $\alpha \geq 1$ and other infinitely divisible probability measures on B-spaces.* Preliminary report, (1987).

[11] Balram S. Rajput. *On the structure of the supports of certain infinitely divisible probability measures on locally convex topological vector spaces.* Submitted to Annals of Probability, (1988).

[12] Balram S. Rajput. *Remarks on the positivity of densities of stable probability measures on R^d.* Preliminary report, (1988).

[13] Balram S. Rajput. *On the supports and densities of probability measures.* To appear in Proceedings of Raj Bose Symposium, (1989).

[14] Ryszard Jajte. *Asymptotic formula for normal operators in non-commutative L_2-spaces.* To appear.

[15] Jan Rosinski. *An application of series representations to zero-one laws for infinitely divisible random vectors.* To appear in Probability in Banach Spaces 7, Progress in Probability, Birkhauser.

[16] Mark Ashbaugh, Balram S. Rajput, Kavi Rama-Murthy and Carl Sundberg. *Remarks on the positivity of densities of stable laws.* In preparation.

[17] Jan Rosinski, Gennady Samorodnitsky and Murad Taqqu. *Sample path properties of stochastic processes represented as multiple stable integrals.* In preparation.